Performance Measures in Data Mining
Common Performance Measures used in Data Mining and Machine Learning Approaches

L. Richter  M. Hecht

Department of Computer Science
Technische Universität München

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Outline
Various algorithms can yield frequent item sets. From frequent item sets \( c \) and \( c \cup \{i\} \) you can derive \( \text{if } c \text{ then } \{i\} \). Typically there is only one item in the RHS (right hand side of the rule).

- **Support (of an item set):**
  \[
  sup(X \rightarrow Y) = sup(Y \rightarrow X) = P(X \land Y)
  \]
  how many times the item set is found in the database

- **Confidence (of a rule):**
  \[
  conf(X \rightarrow Y) = P(Y|X) = P(X \text{ and } Y)/P(X) = sup(X \rightarrow Y)/sup(X)
  \]
Measures for Item Sets

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Measures for Item Sets cont.’d

Measures how frequent an item set / how interesting a rule is in comparison to the expected occurrence (interesting):

- **Leverage (of an item set):**
  
  \[
  lev(X \rightarrow Y) = P(X \text{ and } Y) - (P(X)P(Y))
  \]

- **Lift (of a rule):**
  
  \[
  lift(X \rightarrow Y) = lift(Y \rightarrow X) = \frac{P(X \land Y)}{(P(X)P(Y))} = \frac{conf(X \rightarrow Y)}{sup(Y)} = \frac{conf(Y \rightarrow X)}{sup(X)}
  \]

- **Conviction (of a rule):** Similar to Lift, but directed
  
  Compares the probability that X appears without Y, if they were independent with the observed frequency of X and Y.
  
  \[
  conviction(X \rightarrow Y) = P(X)P(\neg Y)/P(X \land \neg Y) = (1 - sup(Y))/(1 - conf(X \rightarrow Y))
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  \frac{P(X \wedge Y)}{(P(X)P(Y))} = \\
  \frac{\text{conf}(X \rightarrow Y)/\sup(Y)}{\sup(Y)} = \frac{\text{conf}(Y \rightarrow X)/\sup(X)}{
  }
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Supplementary Material

J-Measure

- empirically observed accuracy of rule
- Cross-entropy (measuring how good a distribution approximates another distribution) between the binary variables $\phi$ and $\theta$ with vs. without conditioning on event $\theta$

\[
J(\theta \Rightarrow \phi) = p(\theta) \left( p(\phi|\theta) \log \frac{p(\phi|\theta)}{p(\phi)} + (1 - p(\phi|\theta)) \log \frac{1 - p(\phi|\theta)}{1 - p(\phi)} \right)
\]
Basic Building Blocks for Performance Measures

- True Positive (TP): positive instances predicted as positive
- True Negative (TN): negative instances predicted as negative
- False Positive (FP): negative instances predicted as positive
- False Negative (FN): positive instances predicted as negative

Confusion Matrix:

<table>
<thead>
<tr>
<th></th>
<th>Predicted a</th>
<th>Predicted b</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real a</td>
<td>TP</td>
<td>FN</td>
</tr>
<tr>
<td>Real b</td>
<td>FP</td>
<td>TN</td>
</tr>
</tbody>
</table>
Performance Measures

\[
\text{Accuracy, } acc = \frac{TP + TN}{TP + FN + FP + TN}
\]

\[
= \frac{\text{Number of correct predictions}}{\text{Total number of predictions}}
\]

\[
\text{Error rate, } err = \frac{FN + FP}{TP + FN + FP + TN} = 1 - acc
\]

\[
= \frac{\text{Number of wrong predictions}}{\text{Total number of predictions}}
\]
Performance Measures cont’d

**True Positive Rate, TPR, Sensitivity**

\[ \text{TPR} = \frac{TP}{TP + FN} \]

**True Negative Rate, TNR, Specificity**

\[ \text{TNR} = \frac{TN}{TN + FP} \]

**False Positive Rate, FPR**

\[ \text{FPR} = \frac{FP}{TN + FP} \]

**False Negative Rate, FNR**

\[ \text{FNR} = \frac{FN}{TP + FN} \]
Performance Measures cont’d

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Performance Measures cont’d

True Positive Rate, TPR, Sensitivity = \( \frac{TP}{TP + FN} \)

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Performance Measures cont’d

True Positive Rate, $TPR$, Sensitivity = \( \frac{TP}{TP + FN} \)

True Negative Rate, $TNR$, Specificity = \( \frac{TN}{TN + FP} \)

False Positive Rate, $FPR$ = \( \frac{FP}{TN + FP} \)

False Negative Rate, $FNR$ = \( \frac{FN}{TP + FN} \)
Performance Measures cont’d

**Precision**

\[ p = \frac{TP}{TP + FP} \]

**Recall**

\[ r = \frac{TP}{TP + FN} \]

**F₁ measure**

\[ F₁ \text{ measure} = \frac{2rp}{r + p} = \frac{2 \times TP}{2 \times TP + FP + FN} \]
Performance Measures cont’d

\[Precision, \ p = \frac{TP}{TP + FP}\]

\[Recall, \ r = \frac{TP}{TP + FN}\]

\[F_1 \text{ measure} = \frac{2rp}{r + p} = \frac{2 \times TP}{2 \times TP + FP + FN}\]
Performance Measures cont’d

Precision, $p = \frac{TP}{TP + FP}$

Recall, $r = \frac{TP}{TP + FN}$

$F_1$ measure $= \frac{2rp}{r + p} = \frac{2 \times TP}{2 \times TP + FP + FN}$
Performance Curves

- "costs" of different error types are different
- prediction behaviour changes over the test set
- performance display in 2D
- different domains prefer different chart types
Lift Charts

- from marketing area to evaluate mailing success
- y-axis: number or percentage of responders
- x-axis: sample
- red diagonal: random lift
- green line: optimum lift

Taken from http://www.dmg.org/rfc/pmml-3.2/ModelExplanation.html
ROC Curves

- Receiver Operator Characteristics
- $y$-axis: TPR
- $x$-axis: FPR
- diagonal: random guessing (TPR=FPR)

Taken from http://en.wikipedia.org/wiki/File:Roccurves.png
Sensitivity vs. Specificity

- preferred in medicine
- $y$-axis: TPR
- $x$-axis: TNR (specificity)
- also frequently as ROC curve with 1 - specificity

Taken from http://i.stack.imgur.com/fnUd2.png
Recall Precision Curves

- preferred in information retrieval
- positives are the documents retrieved in response to a query
- true positives are documents really relevant to the query
- y-axis: precision
- x-axis: recall

Taken from http://scikit-learn.github.io/scikit-learn.org/
Error Measures for Regression

Mean – squared error, $MSE = \frac{(p_1 - a_1)^2 + \cdots + (p_n - a_n)^2}{n}$

Root mean–squared error, $RMSE = \sqrt{\frac{(p_1 - a_1)^2 + \cdots + (p_n - a_n)^2}{n}}$

Mean – absolute error, $MAE = \frac{|p_1 - a_1| + \cdots + |p_n - a_n|}{n}$
Error Measures for Regression

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Relative Error Measures

Relative squared error = \[ \frac{(p_1 - a_1)^2 + \cdots + (p_n - a_n)^2}{(a_1 - \bar{a})^2 + \cdots + (a_n - \bar{a})^2} \]

Root relative squared error = \[ \sqrt{\frac{(p_1 - a_1)^2 + \cdots + (p_n - a_n)^2}{(a_1 - \bar{a})^2 + \cdots + (a_n - \bar{a})^2}} \]

Relative absolute error = \[ \frac{|p_1 - a_1| + \cdots + |p_n - a_n|}{|a_1 - \bar{a}| + \cdots + |a_n - \bar{a}|} \]
General Problem

- each algorithm abstracts from observations (instances)
- the aspects kept and the aspect discarded differ between the learning scheme (inductive bias)
- this means also: information about individual instances are contained in the model, too
- individual instance information leads to overfitting
Solution Strategies

- use fresh data, i.e. instances not used for the training
- for very large numbers of instances: simple split in test and training set
- most common: 10-fold cross validation
- LOOCV: Leave one out cross validation